6ELEN018W - Applied Robotics
Lecture 7: Robot Dynamics - Motion upon Forces - Part II

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Last Lecture - Newtonian and Lagrangian Mechanics

- Calculate the dynamic equations of motion for a robot, subject to (generalised) forces, i.e. linear forces and angular forces (torques), one can use either Newtonian or Lagrangian mechanics.

Robot Manipulator Rigid Body Equations of Motion

Consider serial-link manipulator and the motor which actuates each joint j, $j \in \{0, \dots, N\}$.

- The inertia that the motor experiences is a function (depends) of the configuration of the outward links $j_{i+1}, j_{i+2}, \dots, j_N$.
- The equations of motion can be derived using Newton's second law and Euler's equation of rotational motion or the Lagrangian energy-based method.

The actuator forces (joint torques) $\boldsymbol{Q}$ can be written as a set of coupled differential equations:

$$\boldsymbol{Q} = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{f}(\dot{\boldsymbol{q}}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{w} \qquad (1)$$

- $\boldsymbol{q}$ are the joint coordinates (angles)
- $\dot{\boldsymbol{q}}$ are the joint velocities
- $\ddot{\boldsymbol{q}}$ are the joint accelerations
- $\boldsymbol{g}$ is a term which represents the torque due to the gravity acting on the manipulator. This depends only on the configuration (joint angles $\boldsymbol{q}$)

Robot Manipulator Rigid Body Equations of Motion (cont'd)

$$\boldsymbol{Q} = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{f}(\dot{\boldsymbol{q}}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{w}$$

- $\boldsymbol{M}$ is the inertia matrix and depends only on the configuration of the robot (joint angles $\boldsymbol{q}$)
- $\boldsymbol{C}$ is referred to as the Coriolis and centripetal term and this represents the gyroscopic and other forces that act on the robot joints due to the rotation of other robot joints.
- $\boldsymbol{f}$ is the friction force
- $\boldsymbol{J}(\boldsymbol{q})$ is the manipulator Jacobian, and $\boldsymbol{w} \in \mathbb{R}^6$ is the wrench (i.e. forces and torques) applied at the end-effector.

$\Longrightarrow$ This is the **inverse dynamics problem**:
Given the motion find the torques: $(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) \rightarrow \boldsymbol{Q}$

The Newton-Euler Recursive Formula

Solve the equations of motion for the robot serial-link manipulator.
How it works?

- Determine the translational and rotational velocity and acceleration for the centre of mass of each link.
  - Use Netwon's second law for translational motion.
  - Use Euler's law for rotational motion.
- Start at the base of the robot and work outwards to:
  - Determine the translational and angular velocity of the centre of mass for each link in turn.
- Once we reach the end of the robot: start at the tip and work inwards:
  - Determine the force and moment each link exerts on the inboard link

Using the Robotics Toolbox for the Newton-Euler Recursive Formula

```
puma = models.DH.Puma560() # 6-joint robot

zero = np.zeros(6)

# print nominal configuration
print(puma.qn)

puma.plot(puma.qn)

Q = puma.rne(puma.qn, zero, zero)
```

The robot is not moving ($\boldsymbol{q} = 0, \dot{\boldsymbol{q}} = 0$), therefore these torques must be those required to hold the robot up against gravity.
Without gravity:

```
Q = puma.rne(puma.qn, zero, zero, gravity=[0, 0, 0])
```

Using the Robotics Toolbox for the Newton-Euler Recursive Formula

Consider now a case where the robot is moving, joint 1 has a velocity of 1 rad/s^{-1}. In the absence of gravity, the required joint torques are:

```
puma.rne(puma.qn, [1, 0, 0, 0, 0, 0], zero, gravity=[0, 0, 0])
```

The torque on joint 0 is that needed to overcome friction which always opposes the motion. The nonzero torques need to be exerted on the joints to oppose the gyroscopic torques that joint 0 motion is exerting on those joints.

Gravity and Payload

The equations of motion (1) of the serial link robot manipulator:

$$\boldsymbol{Q} = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \underline{\boldsymbol{g}(\boldsymbol{q})} + \boldsymbol{f}(\dot{\boldsymbol{q}}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{w}$$

- Gravity is the force that acts on the robot even if it's not moving.
- The torque that counteracts gravity and stops the arm from collapsing under its own weight.

Payload

A robot needs has to carry an object and place it somewhere else. This is the end-effector's payload.

- The last link in the chain of the robot has to hold the payload.
- This propagates down the chain towards the base of the robot.
- All joints of the robot need to help hold up the payload to stop it being pulled down by the force of gravity.

Effect of the payload:

- As the mass of the object (payload) increases:

$\Longrightarrow$ *One joint will hit its torque limit it will become overloaded. And that's the maximum payload that the robot can hold*.

- The maximum payload of the robot is a function of the torque capabilities of the motors but it is also a function of the configuration (angles) of the robot links.

Calculating the Gravity load (torques) Using the Robotics Toolbox

```
Q = puma.gravload(puma.qn)

# The default gravitational force in Earth
puma.gravity

# Gravity in moon:
print(puma.gravity / 6)

# Place the robot in moon
puma.gravity = puma.gravity / 6

# In moon the torques required are reduced:

print(puma.gravload(puma.qn))
```