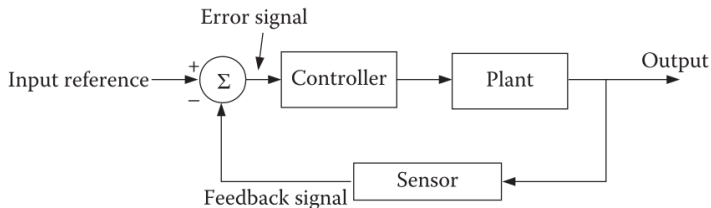


# 5ELEN018W - Robotic Principles

## Lecture 7: Control - Part 2

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# Feedback Controllers



- ▶ Many types of controllers and control algorithms
- ▶ The simplest (and one of the most commonly used for certain systems in industrial process control) is *PID* control

# PID Controllers

## **Proportional + Integral + Derivative**

- ▶ Can have just the P component
- ▶ or both P and I (PI)
- ▶ or both P and D (PD)
- ▶ or all 3 of them (PID)

# Proportional Control (P)

The controller applies an action which is proportional to the error  $e(t)$ :

$$action = K_p \cdot e(t) \quad (1)$$

where  $K_p$  is the gain of the proportional controller.

## A Simple Example: A Robot following a Path

Consider a robot that we would like to keep following a given path at a distance of  $d_1$ .

- ▶ The error at any time  $t$  is measured by  $e(t) = d_1 - d$  where  $d$  is the minimum current distance from the given path.
- ▶ if  $e(t) > 0$ : the action would be turn by  $K_p \cdot e$  degrees away from the path.
- ▶ if  $e(t) < 0$ : the action would be turn by  $K_p \cdot e$  degrees towards the path.

# Integral Control (I)

Proportional control ( $P$ ) might not be able to extinguish a steady-state error.

- ▶ The integral error calculates the accumulated error over time, i.e. it introduces some memory for the error.

$$\text{integral error} = K_i \cdot \int_0^t e(\tau) d\tau \quad (2)$$

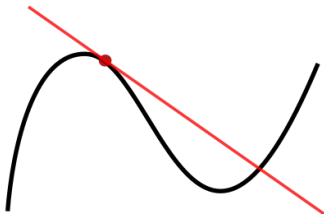
where  $K_i$  is the integral gain.

→ The integral error helps in reducing the steady-state error, but it can lead to overshooting.

# Derivative Control - What is a Derivative?

The derivative of a function measures the sensitivity of changes to the output (value) of the function, based on changes of the input (independent variable) of the function.

- ▶ The slope of the tangent line is equal to the derivative.



→ It is also used to show the direction we need to follow and the magnitude of the step we need to take, in order to reduce an error (machine learning, etc).

## Derivative Control (D)

It takes into account how fast the error changes.

$$\text{derivative error} = K_d \cdot \frac{de(t)}{dt} \quad (3)$$

- ▶ It can correct some of the problems introduced by the integral error, e.g. saturation (the real life physical mechanisms can never be linear - a motor has always upper bounds, how fast it can rotate, etc)

→ It reduces overshooting.



# PID Control

Combining 2 or all 3 components (individual gains can be set to 0 to leave out one or two components from  $I, D$ ).

$$action = K_p \cdot e(t) + K_i \cdot \int_0^t e(\tau) d\tau + K_d \cdot \frac{de(t)}{dt} \quad (4)$$

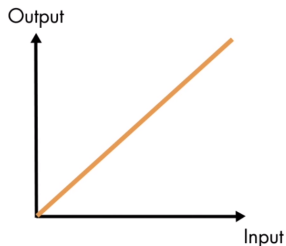
# PID Tuning

A difficult task is to find an appropriate set of the gain parameters  $K_p$ ,  $K_i$ ,  $K_d$  of the controller so as to perform satisfactorily (or optimise) with respect to:

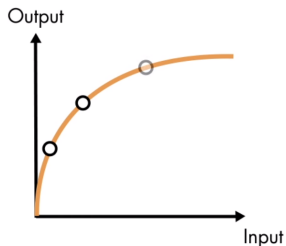
- ▶ steady-state error
- ▶ transient response
- ▶ overshooting
- ▶ settling time

# Linear vs Nonlinear Systems

## Linear System



## Nonlinear System



- ▶ In real life all systems are nonlinear, however many of them can be linearised about their operation point.

Linear systems are easier to analyse and prove mathematically their behaviour and properties.

# Discrete vs Continuous Dynamic Systems

- ▶ Discrete dynamic systems are described by difference equations.

$$x(n) = 5 * x(n - 1) + 6 * x(n - 2) + 2$$

- ▶ Continuous dynamic systems are described by differential equations.

$$\ddot{x} = 5 * \dot{x} + 10 * x + 10$$

# The Laplace Transform

*Linear differential equations* describing physical dynamic systems (including robots) can be transformed to algebraic equations which can be more easily solved (and also analysed) using the Laplace transform.

- ▶ The robotic arm performing a surgery (mass spring damper example), seen last week:

$$m\ddot{x} + b\dot{x} + kx = f \quad (5)$$

assuming all initial conditions are set to 0, then its Laplace transform is:

$$ms^2 + bs + k = F \quad (6)$$

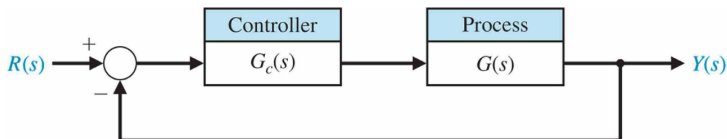
# Transfer Functions

The transfer function of a *linear, time-invariant* system is defined as the ratio of the Laplace transform of the output variable  $Y(s)$  to the Laplace transform of the input variable  $R(s)$ , with all initial conditions assumed to be 0:

$$G(s) = \frac{Y(s)}{R(s)} \quad (7)$$

- ▶ These are used to make easier the modelling and analysis of dynamic systems.

# Feedback (Closed-Loop) Controllers Transfer Function



The transfer function of the closed-loop system is:

$$T(s) = \frac{G(s)G_c(s)}{1 + G(s)G_c(s)} \quad (8)$$

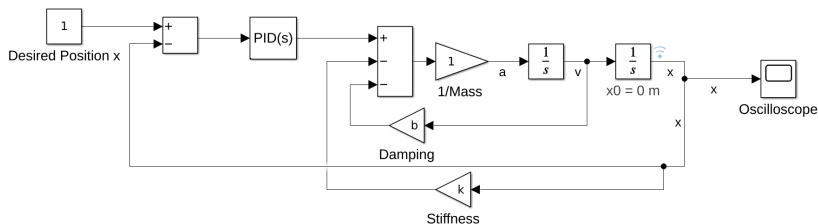
- ▶ A system is unstable where the closed loop transfer function diverges for  $s$  (e.g. where  $G(s)G_c(s) = -1$ ).
- ▶ Stability is guaranteed when  $G(s)G_c(s) < -1$ .

# PID Control for the Robot Arm Surgeon

$$m\ddot{x} + b\dot{x} + kx = f \quad (9)$$

The desired position is 1, starting at position  $x = 0$ . For all the simulations, the following parameters were used:

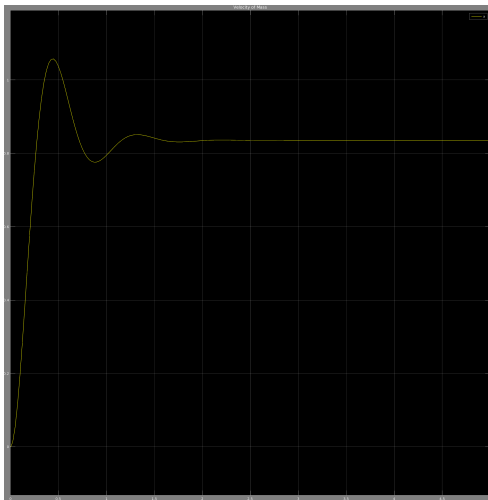
$$m = 1, b = 6, k = 9.86960.$$





# P-Controller

$$K_p = 50$$

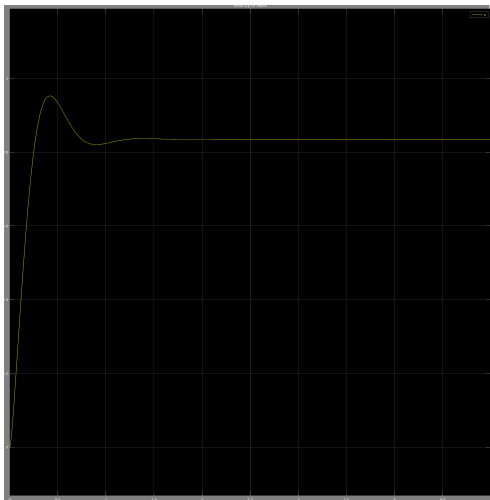


## *P*-Controller (cont'd)

- ▶ large steady-state error
- ▶ large overshoot
- ▶ large settling time

# PD-Controller

$$K_p = 50, K_d = 2.5$$



## *PD*-Controller (cont'd)

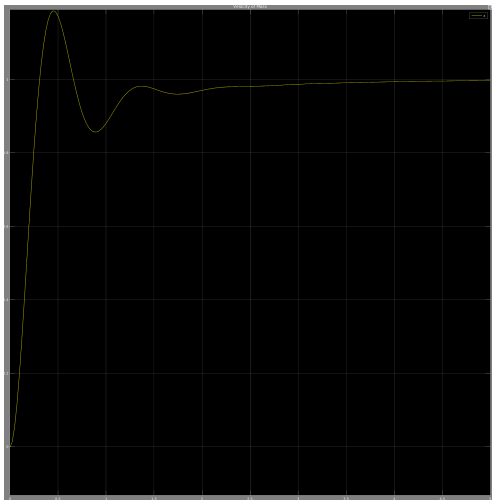
The *D*-component has reduced:

- ▶ the overshoot
- ▶ the settling time

Still large steady-state error.

# PI-Controller

$$K_p = 50, K_i = 40$$

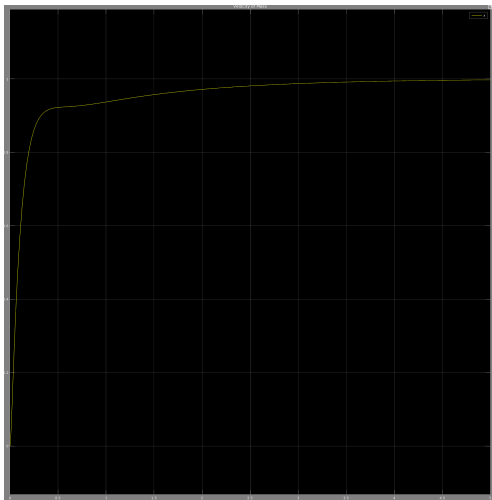


# PI-Controller

- ▶ 0 steady-steady error
- ▶ still large overshoot

# PID-Controller

$$K_p = 50, K_i = 40, K_d = 8$$



## *PID-Controller (cont'd)*

- ▶ no steady steady error
- ▶ no overshoot
- ▶ faster rise time