

# 5ELEN018W - Robotic Principles

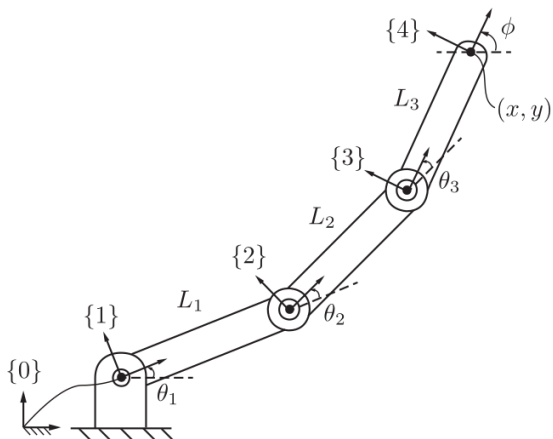
## Lecture 7: Kinematics

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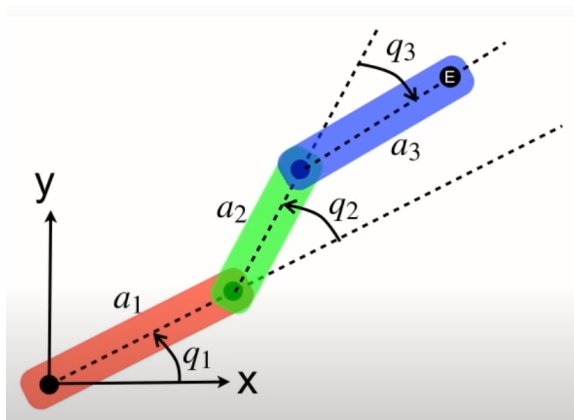
# Forward Kinematics vs Inverse Kinematics

- ▶ *Forward Kinematics*: the calculation of the position and orientation of a robot's end-effector from its joint coordinates  $\theta$ .
- ▶ *Inverse Kinematics*: given a position and orientation of a robot's end-effector, calculate the angles  $\theta$  of the joints.

# Forward Kinematics



# Forward Kinematics



# Representation of Configuration Space of a Robot

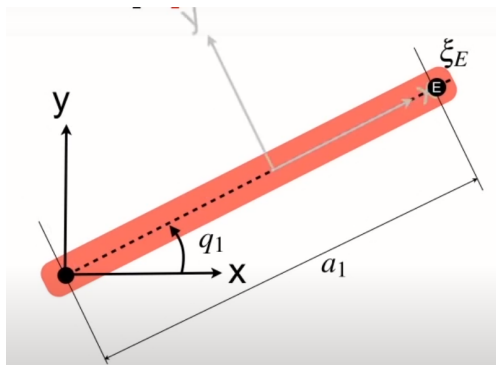
The position and orientation of all links.

We have seen that this can be described with basic transformation matrices that can be multiplied together to get the homogeneous matrix.

$$\text{HomogeneousMatrix} = \text{Transf}_1 * \text{Transf}_2 * \text{Transf}_3 * \dots * \text{Transf}_n \quad (1)$$

where  $n$  is the number of links (assuming that each of these matrices is the total transformation for each link).

## Example of a 1-joint Robot Arm



Rotation by angle  $q_1$  and then translation by  $a_1$ .

The homogeneous transformation describing the overall result can be calculated using the following:

$$\text{EndEffector} = \text{Rot}(q_1) \cdot T_x(a_1) \quad (2)$$

## Example of a 2-joint Planar Robot Arm

A 2-joint Robot Arm in a 2D Plane:

1. Rotation by angle  $q_1$
2. Translation by  $a_1$
3. Rotation by angle  $q_2$
4. Translation by  $a_2$

The homogeneous transformation describing the overall result can be calculated using the following:

$$EndEffector = Rot(q_1) \cdot T_x(a_1) \cdot Rot(q_2) \cdot T_x(a_2) \quad (3)$$

## Example of a 3-joint Planar Robot Arm

1. Rotation by angle  $q_1$
2. Translation by  $a_1$
3. Rotation by angle  $q_2$
4. Translation by  $a_2$
5. Rotation by angle  $q_3$
6. Translation by  $a_3$

The homogeneous transformation describing the overall result can be calculated using the following:

$$EndEffector = Rot(q_1) \cdot T_x(a_1) \cdot Rot(q_2) \cdot T_x(a_2) \cdot Rot(q_3) \cdot T_x(a_3) \quad (4)$$



# The Denavit-Hartenberg (DH) Notation

Attach a coordinate frame to the end of each link.

- ▶ Reduces the relationship between 2 coordinate frames from 6 parameters to 4 parameters.
- ▶ Each joint in a robot is described by 4 parameters.

*How is this achieved?*

The coordinate frames have constraints.

- ▶  $x$  axis of frame  $j$  intersects the  $z$  axis of frame  $j - 1$
- ▶  $x$  axis of frame  $j$  is perpendicular to the  $z$  axis of frame  $j - 1$

→ 6 parameters - 2 constraints means 4 parameters are needed.

## The Denavit-Hartenberg (DH) Notation (cont'd)

4 parameters used associated with each link  $i$  and joint  $i$ :

- ▶  $\theta_i$ : joint angle
- ▶  $r_i$  (or  $a_i$  in most textbooks): link length
- ▶  $d_i$ : link offset
- ▶  $\alpha_i$ : link twist

Each homogeneous transformation  $A_i$  is represented as the product of 4 basic transformations:

$$A_i = Rot_{z,\theta_i} \cdot Trans_{z,d_i} \cdot Rot_{x,\alpha_i} \cdot Trans_{x,r_i} \quad (5)$$

## The Denavit-Hartenberg (DH) Notation (cont'd)

$$A_i = Rot_{z,\theta_i} \cdot Trans_{z,d_i} \cdot Rot_{x,\alpha_i} \cdot Trans_{x,r_i} =$$

$$\begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & 0 \\ \sin(\theta_i) & \cos(\theta_i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_i) & -\sin(\alpha_i) & 0 \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & r_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$
$$\begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & r_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & r_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

# The DH Table

The DH notation requires a table. The number of rows equals the number of joints and it has 4 columns each one corresponding to the 4 parameters for the joint  $i$  of that row.

For example for a robot with 4 joints:

<b>Joint</b>	<b><math>\theta</math></b>	<b><math>r</math></b>	<b><math>d</math></b>	<b><math>\alpha</math></b>
1	$\theta_1$	$r_1$	$d_1$	$\alpha_1$
2	$\theta_2$	$r_2$	$d_2$	$\alpha_2$
3	$\theta_3$	$r_3$	$d_3$	$\alpha_3$
4	$\theta_4$	$r_4$	$d_4$	$\alpha_4$

## Example of DH Notation

Consider the following DH table:

<b>Joint</b>	$\theta$	$r$	$d$	$\alpha$
1	$\pi$	5	2	$\frac{\pi}{2}$

What is the DH matrix which corresponds to the above table?

*Answer:*

$$\begin{matrix} -1.0000 & -0.0000 & -0.0000 & -5.0000 \\ 0.0000 & -0.0000 & 1.0000 & 0.0000 \\ 0 & 1.0000 & 0.0000 & 2.0000 \\ 0 & 0 & 0 & 1.0000 \end{matrix}$$

To calculate, apply Equation (6).

## Finding the Pose of the End-Effector relative to the Base Frame

Assume that  $A_1, A_2, A_3 \dots A_n$  are the DH matrices of all the robot joints  $1, 2, 3, \dots n$ .

Then the calculation requires the multiplication of all the matrices:

$$Pose_{end\_effector} = A_1 \cdot A_2 \cdot A_3 \dots A_n \quad (7)$$

## Example: Calculation of the Pose of the End-Effector

The following DH matrices correspond to the joints of a robot, from robot base to end-effector. Find the pose of the end-effector relative to the robot base.

$$A_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1 & 0 & 0 & -2 \\ 0 & -0 & 1 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Example: Calculation of the Pose of the End-Effector (cont'd)

Simply calculate  $A_1 * A_2 * A_3$ .